LAWS OF MOTION

INERTIA

A body cannot change its state of rest or uniform motion along a straight line. This property is called inertia. Inertia has no unit and no dimension.

INERTIA
- Inertia of Rest
- Inertia of Motion
- Inertia of Direction

- Inertia of Rest
Inability to change state of rest by itself.
- Inertia of Motion
Inability of a body to change its state of uniform motion by itself.
- Inertia of Direction
Inability of a body to change direction of motion by itself.

Newton's Second Law

F_{net} = Rate of change of linear momentum.

Instantaneous: $\vec{F} = \dfrac{d\vec{p}}{dt}$

Average: $\vec{F}_{av} = \dfrac{\Delta\vec{P}}{\Delta t} = \dfrac{P_f - P_i}{\Delta t}$

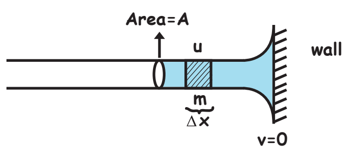
MOMENTUM

$\vec{P} = m\vec{v}$

-It is a vector quantity having direction same as that of velocity

-Unit is kg m/s.

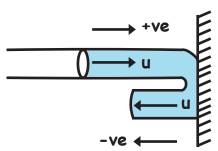
LIQUID JETS



$F_{jet} = \dfrac{P_f - P_i}{\Delta t} = \dfrac{mx0 - mu}{\Delta t} = \dfrac{-mu}{\Delta t} = -\rho A u^2$

$F_{wall} = \rho A u^2$ $\quad (m = \rho A \Delta x,\ \dfrac{\Delta x}{\Delta t} = u)$

When liquid jet bounce back

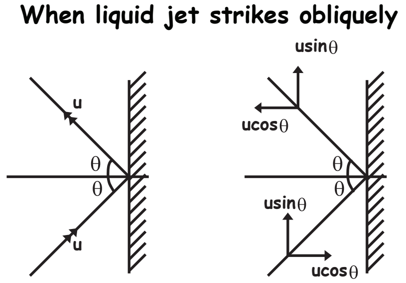


$F_{jet} = \dfrac{P_f - P_i}{\Delta t} = \dfrac{-2mu}{\Delta t}$

$F_{jet} = -2\rho A u^2$

$F_{wall} = 2\rho A u^2$

When liquid jet strikes obliquely



$F_{jet} = -2\rho A u^2\cos\theta$ $\quad F_{wall} = 2\rho A u^2\cos\theta$

Change in momentum=-2mu cos θ

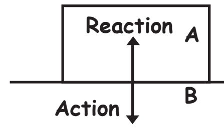
NEWTON'S THIRD LAW

-To every action, there is always an equal (in magnitude) and opposite (in direction) reaction.

- Forces in nature always occurs in pairs.
- A single isolated force is not possible.
- Counter force experienced by a body- reaction
- Action and reaction never act on the same body

* Force exerted on body A by body B (action)
* force exerted on body B by body A (reaction)

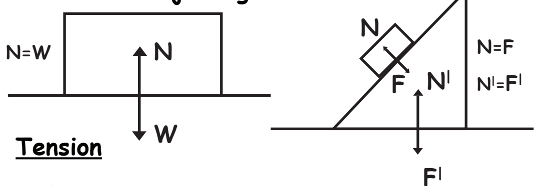
$\vec{F}_{Action} = -\vec{F}_{Reaction}$



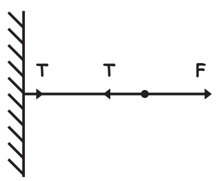
Normal Reaction

- Occurs when two surfaces are in contact with each other

Always perpendicular to the surface and is self-adjusting.

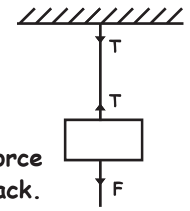


Tension



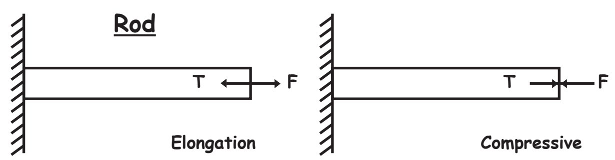
Restoring force developed when a longitudnal force is applied on a body

Ideal Rope



*Massless
*Tension is same everywhere
*Can support only elongation
*Cannot support compressive force
*On compression it becomes slack.
*Tension always acts away from the object.

Rod



can support both elongation and compression

SINGLE BLOCK

Horizontal Force

Acceleration is along x-axis only

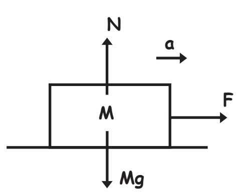
Along y-axis a=0,

N - Mg = Ma (Ma=0)

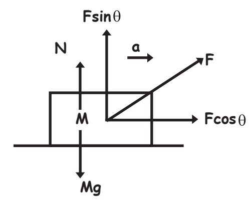
N = Mg

Along x-axis

F - 0 = Ma $\quad a = \dfrac{F}{M}$



Inclined Forces



- If, Fsinθ <Mg block remains in contact with ground
- If, Fsinθ =Mg block just leaves contact with ground
- If, Fsinθ >Mg the block leaves contact with ground and it begins to accelerate.

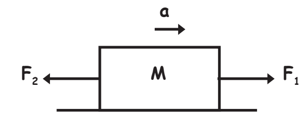
MOTION OF CONNECTED BODIES

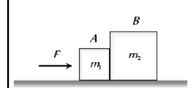
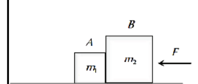
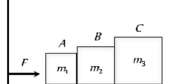
$F_1 > F_2$

$F_1 - F_2 = Ma$

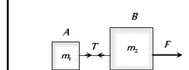
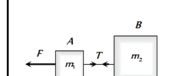
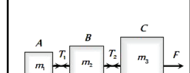
F = Ma

$a = \dfrac{F_{net}}{M}$



Condition	Free body diagram	Equation	Force and acceleration
		$F - f = m_1 a$	$a = \dfrac{F}{m_1 + m_2}$
		$f = m_2 a$	$f = \dfrac{m_2 F}{m_1 + m_2}$
		$f = m_1 a$	$a = \dfrac{F}{m_1 + m_2}$
		$F - f = m_2 a$	$f = \dfrac{m_1 F}{m_1 + m_2}$
		$F - f_1 = m_1 a$	$a = \dfrac{F}{m_1 + m_2 + m_3}$
		$f_1 - f_2 = m_2 a$	$f_1 = \dfrac{(m_2 + m_3)F}{m_1 + m_2 + m_3}$
		$f_2 = m_3 a$	$f_2 = \dfrac{m_3 F}{m_1 + m_2 + m_3}$

MOTION OF BLOCKS CONNECTED BY MASSLESS STRING

Condition	Free body diagram	Equation	Tension and acceleration
		$T = m_1 a$	$a = \dfrac{F}{m_1 + m_2}$
		$F - T = m_2 a$	$T = \dfrac{m_1 F}{m_1 + m_2}$
		$F - T = m_1 a$	$a = \dfrac{F}{m_1 + m_2}$
		$T = m_2 a$	$T = \dfrac{m_2 F}{m_1 + m_2}$
		$T_1 = m_1 a$	$a = \dfrac{F}{m_1 + m_2 + m_3}$
		$T_2 - T_1 = m_2 a$	$T_1 = \dfrac{m_1 F}{m_1 + m_2 + m_3}$
		$F - T_2 = m_3 a$	$T_2 = \dfrac{(m_1 + m_2)F}{m_1 + m_2 + m_3}$

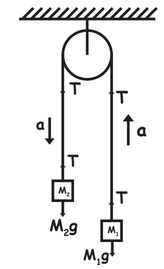
PULLEY

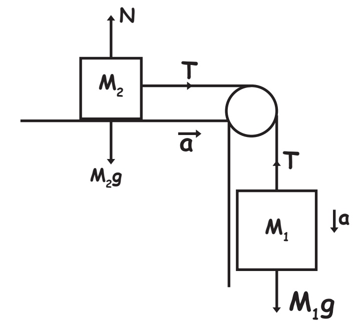
Ideal

$M_1 > M_2$

$a = \dfrac{F}{M} = \dfrac{M_1 - M_2}{M_1 + M_2} g$

$T = \dfrac{2M_1 M_2}{M_1 + M_2} g$

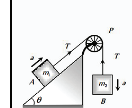
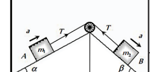
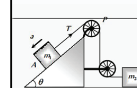




$a = \dfrac{M_1 g}{M_1 + M_2}$

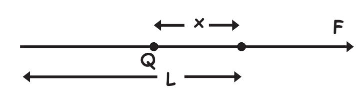
$T = \dfrac{M_1 M_2}{M_1 + M_2} g$

INCLINED PLANE + PULLEY

		$m_1 a = T - m_1 g\sin\theta$	$a = \left[\dfrac{m_2 - m_1\sin\theta}{m_1 + m_2}\right] g$
		$m_2 a = m_2 g - T$	$T = \dfrac{m_1 m_2 (1 + \sin\theta)}{m_1 + m_2} g$
		$T - m_1 g\sin\alpha = m_1 a$	$a = \dfrac{(m_2\sin\beta - m_1\sin\alpha)}{m_1 + m_2} g$
		$m_2 a = m_2 g\sin\beta - T$	$T = \dfrac{m_1 m_2 (\sin\alpha + \sin\beta)}{m_1 + m_2} g$
		$m_1 g\sin\theta - T = m_1 a$	$a = \dfrac{m_1 g\sin\theta}{m_1 + m_2}$

THICK ROPE

Tension will be different at different points.



Mass per unit length $= \dfrac{M}{L}$

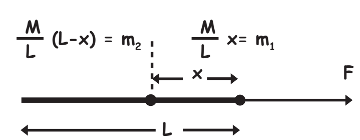
Mass of x length of rope $= \dfrac{M}{L} x$

Note :

Mass of given length $= \dfrac{\text{total mass}}{\text{total length}} \times$ given length

$\dfrac{\text{mass}}{\text{length}}$ = constant

$T = \dfrac{m_2 F}{M}$



LIFT PROBLEMS

Apparent weight of body in a lift

Reading of weighing machine = reaction force exerted by weighing machine (N)

Apparent weight ,($W_{apparent}$) = Reaction force (R)

Case 1: Lift is at rest

R = mg , $W_{apparent} = W_{actual}$ = mg

Case 2: Lift moving up or down with constant velocity

R = mg , $W_{apparent} = W_{actual}$ = mg

Case 3: Accelerated upward at a rate of 'a'

R = m(g+a) = $W_{apparent}$

$W_{apparent} > W_{actual}$ → Feels over weight

Accelerated upward at a rate of 'g'

R -mg = mg , R = 2mg , $W_{app} = 2 \times W_{act}$

Case 4: Accelerated downward at a rate of 'a'

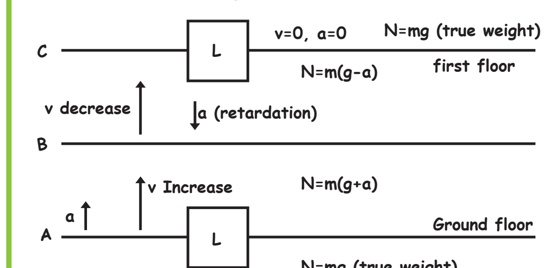
mg - R = ma , R = m(g-a) = W_{app} , $W_{app} < W_{act}$

Accelerated downward at a rate of 'g' [Freefall]

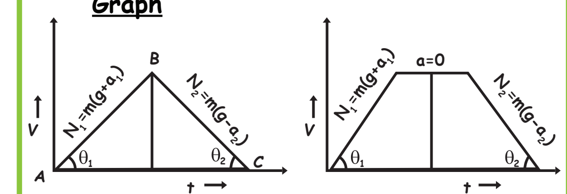
mg - R = mg , R = mg - mg =0 , W_{app} = 0

If a > g : body looses contact with the weighing machine and R becomes zero

Lift moves from ground floor to first floor



Graph



Acceleration, $a_1 = \tan\theta_1$

Retardation, $a_2 = \tan\theta_2$

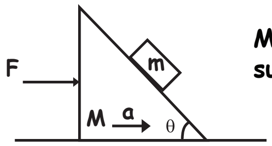
FRAME OF REFERENCE & PSEUDO FORCE

Frame of Reference

A frame in which and observer is situated and makes his observation

Inertial frame of reference	Non-Inertial frame of reference
At rest or moving with uniform velocity along straight line. i.e unaccelerated	Accelerated frame of reference.
Holds Newton's law of motion	Newton's law of motion not applicable.

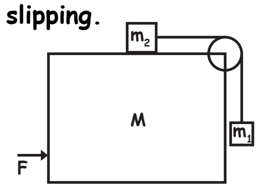
RELATIVE SLIPPING



Minimum force required to push the inclined plane such that "m" does not slip with respect to "M"

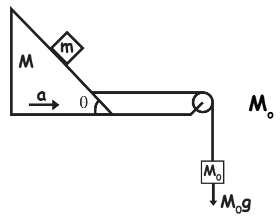
$F = (m+M)\,g\tan\theta$, $a = g\tan\theta$

Minimum acceleration "M" must be pushed such that there is no relative slipping.



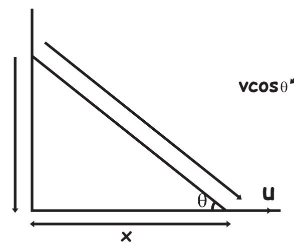
$a = \left(\dfrac{m_1}{m_2}\right) g$

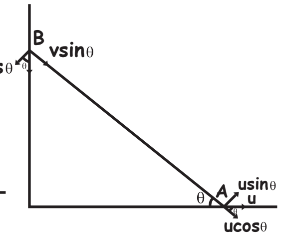
Minimum mass such that there is no relative slipping



$M_0 = \dfrac{M+m}{\cot\theta - 1}$

ROD SLIDING ON A WALL





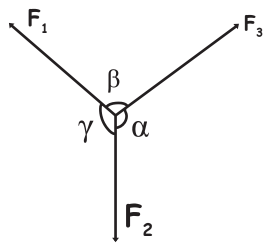
$u = -\left(\dfrac{y}{x}\right)\times(-v)$, $u = v\tan\theta$

velocity of B towards A = $v\sin\theta$

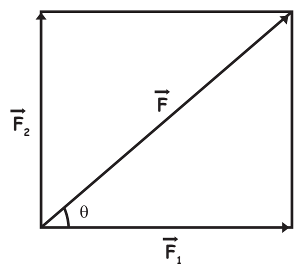
velocity of A away from B = $u\cos\theta$

EQUILLIBRIUM & LAMI'S THEOREM

$\dfrac{F_1}{\sin\alpha} = \dfrac{F_2}{\sin\beta} = \dfrac{F_3}{\sin\gamma}$

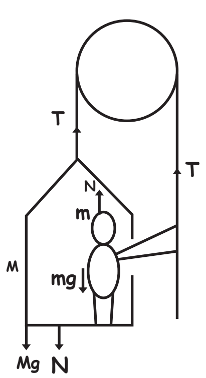


PARALLELOGRAM LAW



$\vec{F} = \vec{F_1} + \vec{F_2}$, $F = \sqrt{F_1^2 + F_2^2}$, $\tan\theta = \dfrac{F_2}{F_1}$

MAN-CAGE PROBLEM



$T = \left(\dfrac{m+M}{2}\right) g$

$N = \left(\dfrac{m-M}{2}\right) g$

IMPULSE

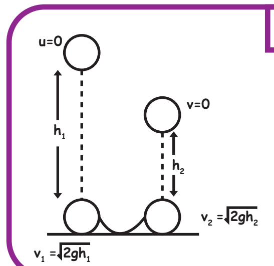
Large force acting for short period of time, As a result of impulse there will be a sudden change in momentum

$F_{ip} = \dfrac{dp}{dt}$

$P_f - P_i = \int F_{ip}\,dt$ = area under F-t graph

I = Impulse = $P_f - P_i = \int_0^t F_{ip}\,dt$ = area of F-t graph

Case-1



Impulse

$I = P_f - P_i$

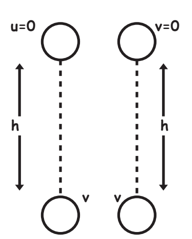
$= mV_2 - (-mV_1)$

$I = m(V_1 + V_2)$

Average impulsive force

$F_{avg} = \dfrac{\text{impulse}}{t_0} = \dfrac{\Delta P}{t_0}$

Case-2



Impulse

$m(v+v) = J$

$= m(2v)$

$J = 2mv$

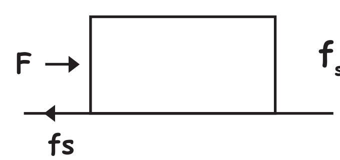
Average impulsive force

$F_{avg} = \dfrac{\Delta P}{t_0} = \dfrac{2mv}{\Delta t}$

FRICTION

Static friction

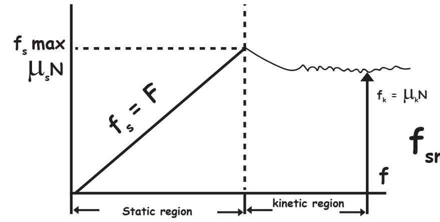
- It is a self adjusting force.
- The opposing force that comes into play, when object tends to move over the surface of other object, but the actual motion has not yet started.
- As applied force increases static friction also increases.
- If the applied force is increased than the force of static friction also increases.
- The body doesn't move untill maximum value of static friction is attained
- The value is called limiting friction or f_{smax}



$f_{smax} = \mu_s N$

Kinetic friction

If the applied force is increased further and sets the body in motion, the friction opposing the motion is called kinetic friction.



$f_{smax} = \mu_k N$

ANGLE OF FRICTION

Angle made by resultant of normal (N) & frictional force(f_s) with normal

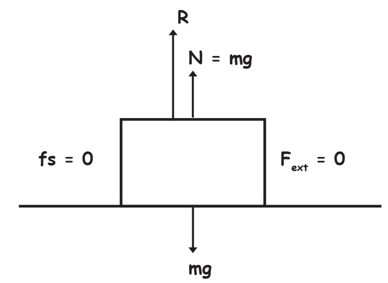
$R = \sqrt{N^2 + f_s^2}$ R is resultant

$\tan\theta = \dfrac{f_s}{N}$

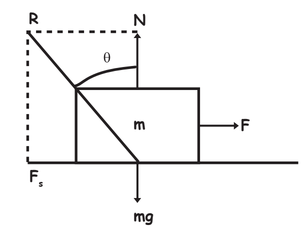
When $f_s = f_{s\,max}$ $R = N\sqrt{1+\mu_s^2}$

$\tan\theta = \mu_s$ μ_s is Coefficient of friction

i) $F_{ext} = 0$



ii) $0 < F_{ext} < f_{smax}$

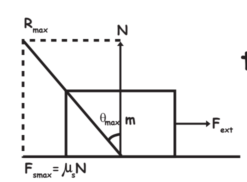


$0 \le \tan\theta \le \mu_s$

$N < R < N\sqrt{1+\mu_s^2}$

iii) $f_{ext} = f_{smax}$

$R_{max} = N\sqrt{1+\mu_s^2}$

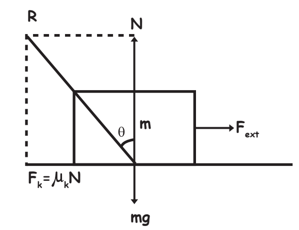


$\tan\theta = \mu_s$

iv) $f_{ext} > f_{smax}$

$R = N\sqrt{1+\mu_k^2}$

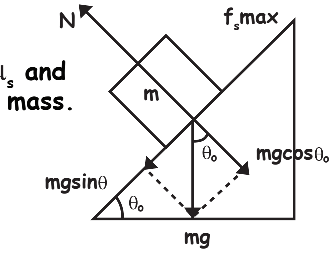
θ = Angle of kinetic friction



ANGLE OF REPOSE

depends only on μ_s and is independent of mass.

$\tan\theta_0 = \mu_s$



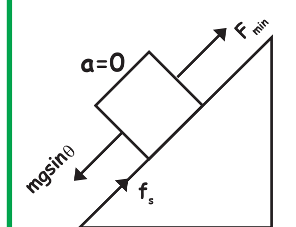
Variation of R

As angle of inclined plane increases, R remains constant and when sliding starts R starts decreasing.

Variation of angle of friction.

As angle of inclined plane increases, angle of friction will also increases and as sliding starts its value becomes constant and $\tan\theta = \mu_k$

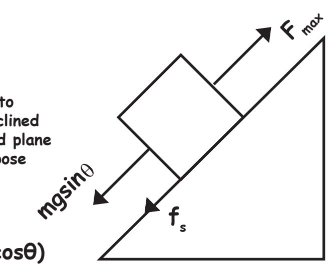
Minimum & Maximum force (applied parallel to inclined plane)



$F_{min} = mg(\sin\theta - \mu_s\cos\theta)$

F_{min} is the minimum force required to make the block stationary if the angle of inclined plane is greater than angle of repose

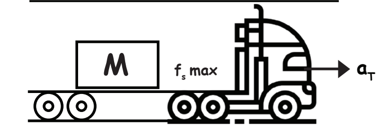
F_{max} is the force required to move the block along the inclined plane if the angle of inclined plane is greater than angle of repose



$F_{max} = mg(\sin\theta + \mu_s\cos\theta)$

$mg(\sin\theta - \mu_s\cos\theta) \le F \le mg(\sin\theta + \mu_s\cos\theta)$

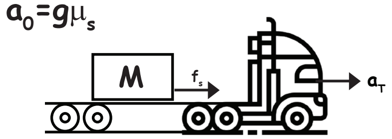
HORIZONTAL TRUCK BOX



$a_0 = g\mu_s$

Case-1

$a_{truck} < a_0 = g\mu_s$

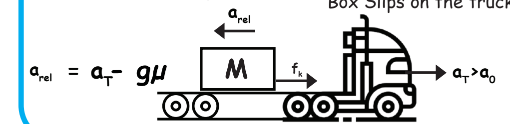


$f_s = Ma_T$ Box does not slip on the truck

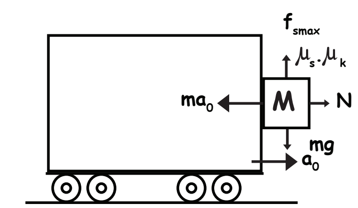
Case-2

$a_{truck} > a_0 = g\mu_s$ Box Slips on the truck

$a_{rel} = a_T - g\mu$

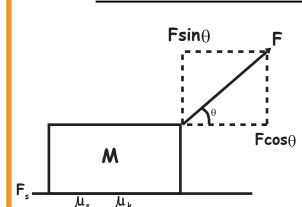


VERTICAL TRUCK BOX



If $a_0 > \dfrac{g}{\mu_s}$ then box does not slip

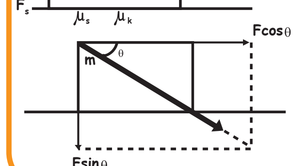
PULLING FORCE & PUSHING FORCE



$F\cos\theta \ge \mu_s(mg - F\sin\theta)$

$F \ge \dfrac{\mu_s mg}{\cos\theta + \mu_s\sin\theta}$

$F_{pulling} = \dfrac{\mu_s mg}{\cos\theta + \mu_s\sin\theta}$



$F\cos\theta \ge \mu_s(mg + F\sin\theta)$

$F \ge \dfrac{\mu_s mg}{\cos\theta - \mu_s\sin\theta}$

$F_{pushing} = \dfrac{\mu_s mg}{\cos\theta - \mu_s\sin\theta}$

LAWS OF MOTION